



# Spatial interactions minimize relative disparity between adjacent surfaces

Zhilei Zhang, Mark Edwards<sup>1</sup>, Clifton M. Schor\*

*School of Optometry, University of California, Berkeley, CA 94720, USA*

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## Abstract

Computational models of stereopsis employ a number of algorithms that constrain stereo matches to produce the smallest absolute disparity and to minimize the relative disparity between nearby features. In some natural scenes, such as large slanted textured surfaces, these two constraints lead to different matching solutions. The current study utilized a stimulus in which there was a large discrepancy in both the magnitude and direction of matches that solved for minimum absolute and minimum relative disparity. This discrepancy revealed a dominance for the minimum relative disparity over the minimum absolute disparity matching solution that increased with spatial proximity, spatial frequency and width of adjacent features. The likelihood of a minimum-relative-disparity matching solution also increased when the difference between the amplitudes of the alternative relative disparities was large. When alternative relative disparity matching solutions had similar amplitudes but opposite signs (crossed vs. uncrossed), an idiosyncratic depth bias served as a tie-breaker. The present results show that absolute disparity matches are constrained to minimize relative disparity between adjacent features. © 2001 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

Stereopsis is the sense of relative depth between two or more features. It is stimulated by differences between absolute disparities subtended by these features (Westheimer, 1979). Absolute disparity of a feature is the difference in the locations of its retinal images in the two eyes from corresponding retinal points. When stimulated, corresponding points produce percepts of identical visual directions (zero disparity). The retinal topography of corresponding points is described by the horopter, which is the locus of points in space that are imaged with zero disparity. Absolute disparity of an object quantifies its retinal disparity relative to the horopter. Stereopsis is stimulated by differences between absolute disparities, i.e. relative disparity. Relative disparity describes the disparity difference between features.

One of the major problems for stereopsis is to determine which binocular retinal images correspond to the same object in space. Solving the so-called correspondence, or matching problem, can be a complex process because it is often possible to match any given feature in one eye with many similar features in the other (Marr & Poggio, 1979). Matching ambiguity is particularly pronounced for natural scenes that contain textured surfaces such as tree foliage. Given these matching ambiguities, the visual system needs to employ a number of heuristics in order to solve the correspondence problem.

The horopter plays an important role in one of these processing heuristics. Stereo matches can reduce the absolute disparity of images subtended by the object. We refer to this approach as the minimum-absolute-disparity rule, and it minimizes the disparity offset from the horopter. It has also been referred to as the nearest-neighbor rule (Arditi, Kaufman, & Movshon, 1981). Another processing heuristic is to minimize relative disparities between *multiple* objects. We refer to this approach as the minimum-relative-disparity rule. It has

\* Corresponding author.

*E-mail address:* schor@socrates.berkeley.edu (C.M. Schor).

<sup>1</sup> Present address: Department of Psychology, Australian National University, Australia.

also been referred to as the nearest-disparity rule (Mitchison & McKee, 1987a,b) and a number of studies have shown that it can influence stereo matches made with ambiguous stimuli (Kontsevich, 1986; Mitchison & McKee, 1987a,b; McKee & Mitchison, 1988; Papathomas & Julesz, 1989). Because the objective of the minimum-relative-disparity rule is to reduce the magnitude of the depth discontinuity between objects, this approach can be thought of as a smoothness or continuity constraint.

The minimum-absolute-disparity and minimum-relative-disparity rules do not necessarily lead to the same stereo matching solutions. The aim of the present study was to determine which of the two rules would take precedence when they had different matching solutions. We achieved this aim by using stimuli that pitted matching solutions based on the minimum-absolute-disparity rule and minimum-relative-disparity rule against one another to determine which one would take precedence. Specifically, we used a stimulus configuration composed of three non-overlapping periodic stimuli (Gabor patches) presented in two disparity planes (see Fig. 1). Disparity magnitudes of the stimuli were selected such that if matches minimized the absolute disparities of all the Gabors, then their relative disparity would not be minimized; if matches minimized the relative disparity difference between the Gabors, then

the absolute disparity of all the Gabors would not be minimized. Thus, the visual system had the alternative of either minimizing all absolute disparities or minimizing the relative disparity. Matching priorities with both transient and sustained stimuli were examined (Schor, Edwards, & Pope, 1998). In this study, the phase (deg) of the sine-wave carrier of the Gabor was the metric used to quantify disparity except when describing disparity between different carrier spatial frequencies. In the latter case, the metric was positional disparity (arc min).

## 2. Experiment 1: The influence of spatial factors on depth ordering

The likelihood that minimum-relative-disparity matches supersede minimum-absolute-disparity matches appears to increase as both the spatial frequency and the width of the stimulus are increased (Edwards & Schor, 1999). The aim of this experiment was to establish the optimal values of these two stimulus parameters for applying the minimum-relative-disparity rule. Because it appears that the likelihood of applying the minimum-relative-disparity rule increases as the temporal duration of the stimulus is decreased (Mitchison & McKee, 1987a,b), only the transient system was tested in this experiment.

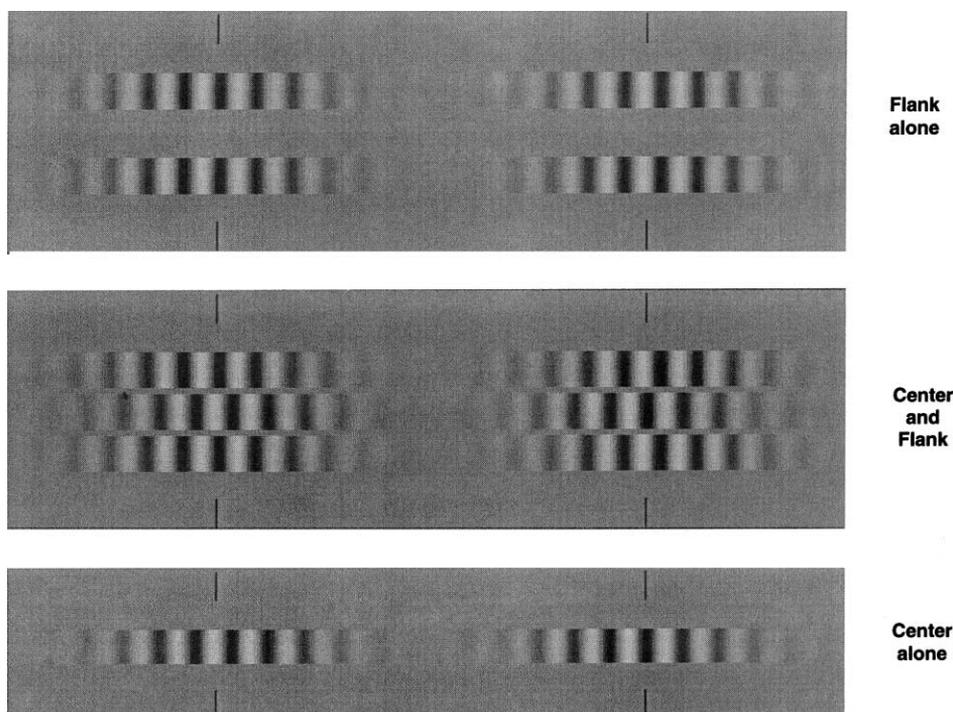
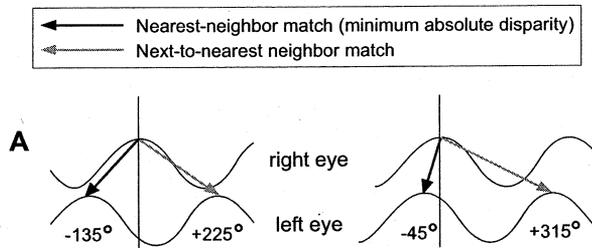


Fig. 1. Stereogram illustrating the center and flanking Gabor stimuli used in the experiments. Center and flanking Gabors subtend equal disparities of opposite sign. The center stimulus in the middle panel is surrounded by upper and lower flanking stimuli with same disparity value ( $135^\circ$  minimum absolute disparity) but in opposite sign. The flank and center are shown in isolation in the top and bottom panels, respectively. The black vertical fixation lines are aligned at zero disparity. When the top panel is cross-fused, the flank appears nearer than the fixation lines. When the bottom panel is cross-fused, the center appears farther than the fixation lines. When the center panel is cross-fused, both the center and flanks appear either nearer or farther than the fixation lines, and the depth ordering is reversed compared to the top and bottom panel.

### Illustration of nearest-neighbor and next-to-nearest-neighbor match solutions for a single target



### Illustration of minimum absolute and minimum relative disparity match solutions for two targets

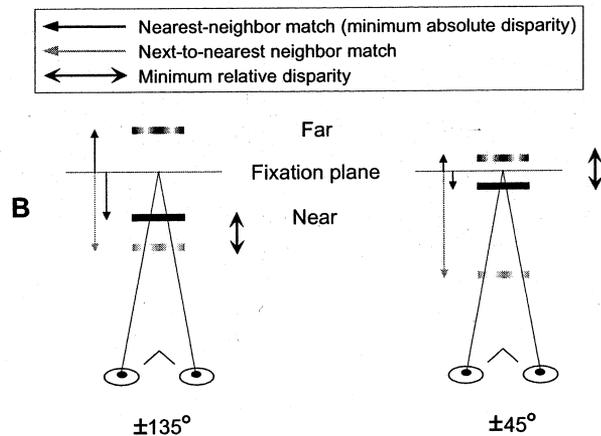


Fig. 2. (A) Schematic example of potential nearest neighbor and next-to-nearest-neighbor matching solutions for a single Gabor. The left column illustrates a nearest-neighbor match of  $-135^\circ$  in the crossed direction and next-to-nearest-neighbor match of  $225^\circ$  in the uncrossed direction. The right column illustrates a nearest-neighbor match of  $-45^\circ$  in the crossed direction and a next-to-nearest-neighbor match of  $315^\circ$  in the uncrossed direction. Nearest-neighbor matches result in minimum absolute disparity. (B) Top schematic view of the center and flanking Gabors. A center Gabor (dark-dashed bar) subtending an uncrossed disparity is surrounded vertically by two flanking Gabors (black bar) that subtend a crossed disparity. The figure shows a single nearest-neighbor match for the flanking Gabor (black bar) and two possible matches for the center Gabor (dashed bar). The black and dark-dashed bars represent nearest-neighbor matches, and the light-dashed bar represents a next-to-nearest neighbor match of the center Gabor. The left panel illustrates a nearest-neighbor match of the flanks at  $135^\circ$  in the crossed direction. The two possible matches of the center Gabor are a nearest-neighbor match of  $135^\circ$  in the uncrossed direction and a next-to-nearest-neighbor match of  $225^\circ$  in the crossed direction. The relative disparity between the center and flanks is smaller ( $90^\circ$ ) for the next-to-nearest match than for the nearest-neighbor match ( $270^\circ$ ). The right panel illustrates that a nearest neighbor match of both the center (uncrossed) and flanks (crossed) at  $45^\circ$  absolute disparity results in a smaller relative disparity than obtained with a false match of the center Gabor at  $315^\circ$  in the crossed direction.

#### 2.1. Experimental logic

The stimulus used in these experiments was designed

to put the matching solutions based upon minimum-absolute-disparity and minimum-relative-disparity rule into conflict with one another. This was achieved with periodic stimuli. In a periodic stimulus, like a vertical luminance sine-wave grating (i.e. the sine-wave carrier in a Gabor stimulus), each feature in one eye has many potential stereo matches in the other eye (Fig. 2). Two of these potential matches will be considered here. The first matches features that are closest to corresponding retinal-locations (i.e. nearest-neighbor features) to form the minimum absolute-disparity. The second matches the next largest possible absolute disparity (i.e. next-to-nearest-neighbor features). The minimum absolute disparity always results from the nearest-neighbor match. The minimum relative disparity can result from either the nearest-neighbor or next-to-nearest-neighbor match, depending on the stimulus configuration. In the example shown in the left column of Fig. 2A, the nearest-neighbor match results in an absolute disparity of  $135^\circ$  in crossed direction. The next-to-nearest-neighbor match results in an absolute disparity of  $225^\circ$  in the uncrossed direction (i.e.  $360^\circ - 135^\circ$ ). There are two important points to note when comparing these two potential matches. The first is that the next-to-nearest-neighbor match results in a larger absolute disparity, i.e. offset from the horopter is not minimized. The second point is that the two matching solutions result in disparities that are in opposite directions. Given the minimum-absolute-disparity rule, when an isolated grating is presented, observers should make the nearest-neighbor match. The wallpaper illusion demonstrates this minimum-absolute-disparity solution (Brewster, 1844).

Next, consider the situation where two different absolute disparity stimuli are presented (Fig. 2B, left column). Like the example shown in the left column of Fig. 2A, the nearest-neighbor match results in a  $135^\circ$  absolute disparity, with one surface matched with crossed disparity and the other with uncrossed disparity. This matching combination results in a relative disparity of  $270^\circ$  ( $135^\circ + 135^\circ$ ) between the two stimuli. However, if, for example, the uncrossed disparity stimulus that was matched by the nearest neighbor underwent a next-to-nearest-neighbor match instead, then it would have a crossed disparity of  $225^\circ$ . This matching combination would result in a relative disparity of  $90^\circ$  ( $225^\circ - 135^\circ$ ), which is less than the previous relative disparity of  $270^\circ$ . Note that the next-to-nearest-neighbor match would reverse the depth ordering of the two Gabors. Thus, the stereo system either can apply nearest-neighbor matches to all objects, which would result in the minimum absolute disparities for all matches but not the minimum relative disparity between matches, or could apply the next-to-nearest-neighbor match to one target and minimize relative

disparity between targets. Note that in this example, it is not possible to find a common matching solution for both minimum-absolute-disparity and minimum-relative-disparity rules. Either both absolute disparities are minimized, or the relative disparity between the matches is minimized by increasing the absolute disparity of one of the surfaces. The nearest-neighbor match shown in Fig. 2a (right column) is an absolute disparity of  $45^\circ$  in the crossed direction. The next-to-nearest-neighbor match is  $315^\circ$  in the uncrossed direction. When two absolute disparities are presented on either side of the fixation plane, the relative disparity between them is  $90^\circ$  (Fig. 2B, right column). If one of these stimuli undergoes a next-to-nearest-neighbor match ( $315^\circ$  in the crossed direction), the relative disparity will be larger ( $270^\circ$ ). In general, if the absolute disparity of both stimuli were less than  $90^\circ$ , the same matching solution would satisfy both rules.

## 2.2. Method

### 2.2.1. Observers

Three observers were used, two of the authors (ZZ & CS) and an observer (EG), who was naïve with respect to the aims of the experiment. All observers had either normal (EG) or corrected to normal visual acuity, normal stereopsis (as measured by a Randot Stereo test) and no history of any binocular visual-disorders.

### 2.2.2. Stimuli and procedure

The stimulus consisted of three vertically offset, one-dimensional luminance Gabor functions with opposite disparity directions. An example is illustrated in the central panel of Fig. 1. When crossed fused, the central Gabor has an uncrossed disparity, and the flanks have a crossed disparity. The top and bottom panels illustrate Gabor patches that have the same disparity as the flank and center of the central panel, respectively, i.e.  $135^\circ$  in the crossed and uncrossed direction. Notice that the center Gabor in the middle panel appears in front of the crossed-disparity flanks, even though it appears behind the fixation lines when presented without the flanks (bottom panel). Disparities were produced by displacing both the envelope and the carrier of the Gabor equally.

Absolute disparities of  $22.5^\circ$ ,  $33.75^\circ$  and  $135^\circ$  and carrier spatial-frequencies of 0.25 and 1 cpd were employed. The  $33.75^\circ$  absolute disparity of the 0.25 cpd stimulus produced the same absolute disparity in units of minutes of arc ( $22.5$  min arc) as the  $135^\circ$  absolute disparity of the 1cpd stimulus. Because we have previously shown that both the spatial frequency and width of the stimuli can affect the likelihood of reverse-depth perception, these parameters were also varied (Edwards & Schor, 1999). Gabor widths, i.e. standard deviations ( $\sigma$ ) of the Gaussian window, of  $1^\circ$ ,  $2^\circ$ ,  $4^\circ$ ,  $6^\circ$ , and

$7^\circ$  were used. In all conditions, the center and flank stimuli had the same width and spatial frequency, and equal but opposite absolute disparities. The height of each Gabor function was fixed at  $1^\circ$ , and the vertical separation between the Gabors (i.e. the distance between adjacent horizontal edges of the Gabors) was fixed at  $0.5^\circ$ . A center-only condition was employed in which only one Gabor was presented. Its depth was judged relative to the fixation point. This condition was included to determine if reversed depth direction was perceived only when there were multiple depth planes present.

During stimulus presentation, the observer first maintained binocular fixation on a pair of crosses and vertical nonius lines. The observer initiated the presentation of the test stimulus when binocular fixation was established, and the nonius lines were perceived as aligned. The disparity direction of the center Gabor was selected randomly in each trial. The Gabor stimuli and a fixation point were presented for 140 ms in a rectangular temporal-window. This exposure duration was selected because next-to-nearest-neighbor matches appear to be more prevalent at short, rather than long, durations (Mitchison & McKee, 1987a,b). In subsequent studies in this report, we also used a 7 s raised-cosine temporal-window. We have previously shown that these two durations (140 ms and 7 s) and temporal windows (rectangular and raised cosine) selectively drive the transient and sustained stereo-systems, respectively (Pope, Edwards, & Schor, 1999; also see Kumar & Glaser, 1994). When the Gabor stimuli disappeared, the fixation cross and nonius lines reappeared. The observer's task was to indicate the depth direction of the center Gabor relative to the two flanking Gabors. In the center-only condition, the depth direction of the single Gabor was judged relative to the fixation point. Because we are proposing that next-to-nearest-neighbor matches are made only when multiple depth planes are present, we predict that depth matches with the center-only stimulus would range from chance to 100% in the nearest-neighbor direction.

The viewing distance was 0.5 m, the mean luminance of the display when viewed through our apparatus was  $3$  cd/m<sup>2</sup>, and the contrast was 50%. Stimuli were presented in blocks of 20. Each data point reported represents the mean of 10 blocks of trials. The use of a low-contrast stimulus reduced the high-temporal-frequency content of the sustained target and thus minimized the contribution of any transient mechanism.

## 2.3. Apparatus

Stimuli were generated using a Cambridge Research Systems VSG 2/3 graphics card in a host Pentium computer and were displayed on a custom Model 3 Vision Research Graphics monitor. The dichoptic half-

images were selectively presented to each eye via the use of Vision Research Graphics ferro-electric shutters. The fast decay rate of the monitor's P46 phosphor (0.1  $\mu$ s to 10% of the phosphor's initial luminance value) ensured that there was no interocular cross-talk via the shutters. The frame rate of the monitor was 120 Hz so that the effective frame rate to each eye was 60 Hz. At this frame rate, there was no noticeable flicker of the images. The observer initiated each trial and responded via the mouse. A chin rest was used to stabilize the observer's head.

## 2.4. Results and discussion

The results for the three observers are shown in Fig. 3. The left column presents performance with the combined center and flanking Gabor condition, and the right column presents performance with the center-only condition. Stereo-performance is plotted for these conditions as a function of envelope width, i.e. sigma (deg). Performance was quantified in terms of the percentage of responses consistent with all stereo matches in the nearest-neighbor direction (i.e. consistent with the minimum-absolute-disparity rule). Performance levels significantly less than 50% indicate that the perceived depth direction of the center relative to the flanks was reversed compared to percepts with greater than 50% performance and that stereo matches were made in accordance with the minimum-relative-disparity rule. Error bars represent plus and minus one standard error of the mean.

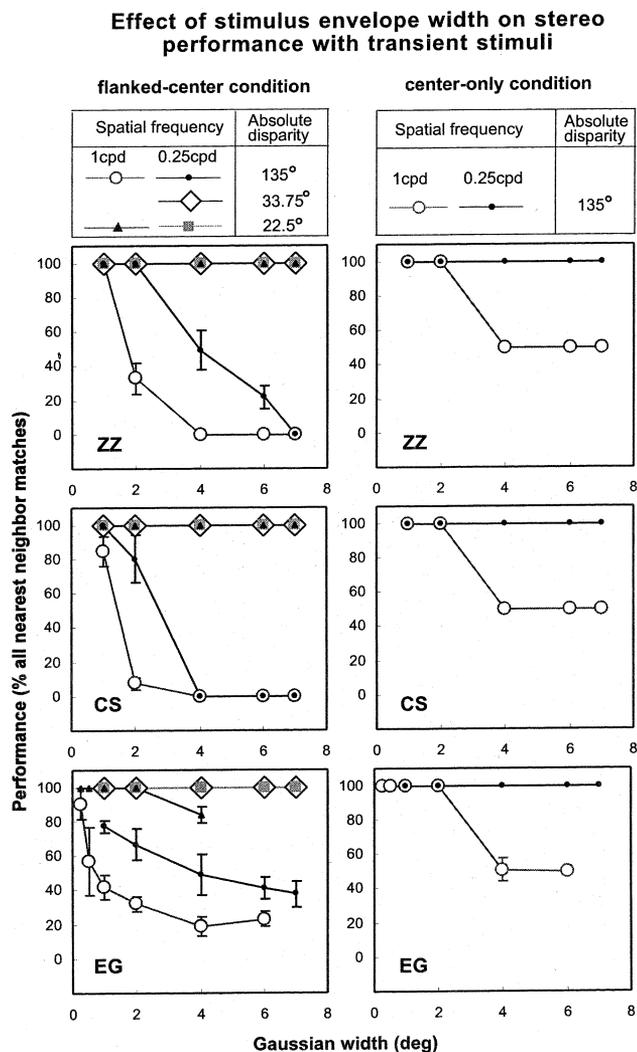


Fig. 3. Results for Experiment 1. Stereo-performance (% all nearest-neighbor matches) is plotted against the Gaussian width (degrees) of the Gabor stimuli at two carrier spatial frequencies (1 cpd and 0.25 cpd) and three absolute disparities (22.5°, 33.75° and 135°). The vertical separation between stimuli was 0.5°. The left panel plots responses to combined center-flanking transient stimuli as a function of width. The right panel plots responses to center-only transient Gabor stimuli with 135° absolute disparity as a function of width. Next-to-nearest-neighbor matches only occur with the combined center-flanking stimuli at 135° absolute disparity.

Inspection of the left column (Fig. 3) illustrates that performance was determined by both absolute disparity amplitude and stimulus width. When absolute disparities were less than 90°, performance was always near 100% (i.e. nearest-neighbor matches). Note that these absolute disparities, 22.5° and 33.75°, result in relative disparities that were less than 180° (i.e. 45° and 67.5°, respectively). When the absolute disparities were greater than 90°, i.e. 135°, performance depended upon stimulus width. Note that absolute disparities greater than 90° result in relative disparities greater than 180°. For small widths, performance was 100%, and it decreased to less than 50% as width increased. For two observers (ZZ & CS), performance reached 0%. The likelihood of making next-to-nearest-neighbor matches that minimized relative disparity also increased as the spatial frequency of the carrier increased. The spatial frequencies of the two carriers differed by a factor of four, but the difference in lateral separation between the corresponding data sets differ by a factor ranging from 2 (CS and ZZ) to four (EG), indicating that a constant bandwidth cannot account for spatial frequency differences. These results are consistent with our earlier findings (Edwards & Schor, 1999).

The right column of Fig. 3 displays results for the center-only condition. Stereo performance for the 135° absolute-disparity condition remained at a level equal to or greater than 50%. That is, reversed depth resulting from next-to-nearest-neighbor matches never occurred with the center-only Gabor, even though they did occur when the same absolute disparity was flanking by two Gabors subtending a different disparity. This finding indicates that next-to-nearest-neighbor matches require at least two depth planes, i.e. relative disparity. The deterioration in stereo performance when the width of the center-only Gabor was increased is likely the result of reduced signal strength (Eagle & Rogers, 1997; Prince & Eagle, 2000). Increasing the width of the

stimulus increases the number of cycles of the carrier and hence increases the number of potential matching solutions. Increasing width also places the unambiguous edges of the Gabor patch at more eccentric retinal locations. The increase in matching ambiguity weakens the effective signal strength of the stimulus.

### 3. Experiment 2: Are matches based on absolute or relative-disparity magnitude?

Our claim is that the type of match made, i.e. nearest neighbor or next-to-nearest neighbor (and hence the perceived depth ordering of the center and flanking Gabors), is determined by a matching preference that minimizes relative, rather than absolute, disparity. Thus, by varying the relative disparity between the flank and center Gabors, we should be able to elicit either nearest-neighbor or next-to-nearest-neighbor matches with the same absolute disparity of the center Gabor.

#### 3.1. Method

##### 3.1.1. Stimuli and procedure

The aim in this experiment was to illustrate the emphasis on relative disparity in the matching process. We manipulated relative disparity by presenting different absolute disparities or different spatial frequencies in the center and flank. In the first condition, we kept the center Gabor at an absolute disparity of  $135^\circ$  and put the absolute disparity of the flanks at  $45^\circ$  with the same sign (direction) as the center. The spatial frequency of both center and flank carriers was 1 cpd, and they were separated by  $0.5^\circ$ . With this configuration, nearest-neighbor matches for all surfaces would result in the smallest relative disparity between the surfaces ( $90^\circ$ ). Thus, unlike the case in the previous experiment, we would expect nearest-neighbor matches to be made with a center Gabor with  $135^\circ$  absolute disparity.

In the second condition, we changed the relative spatial-frequencies of the center and flanking carriers. The carrier of the center Gabor was 1 cpd, and that of the flanks was 0.25 cpd. The center and flanking Gabors had minimum absolute disparities of  $45^\circ$  of opposite sign. With such a configuration, nearest-neighbor matches of all stimuli would result in a relative disparity between the center and flanking Gabors of 37.5 arc min, while making a next-to-nearest-neighbor match of the center Gabor would result in a smaller relative disparity of 22.5 arc min.

#### 3.2. Results and discussion

In condition one, when the center Gabor had an absolute disparity of  $135^\circ$ , it was matched to a nearest

neighbor when the flanking Gabors had an absolute disparity of  $45^\circ$  of the same sign. In condition two, when the spatial frequencies of the center and flanking Gabors were unequal, a next-to-nearest-neighbor match occurred when the minimum absolute disparity of the Gabors was only  $45^\circ$ . These results are not plotted. The matches observed in both the first and second condition resulted in the smallest relative disparity between the center and flanking Gabors. The results of the present experiment show that the likelihood of perceiving reverse depth does not depend upon the absolute disparity of individual targets. Rather, reverse-depth ordering is perceived when the binocular match minimizes the relative disparity between the two depth planes. The results also illustrate that disparity interactions occur across spatial scales of at least 0.25 and 1 cpd.

### 4. Experiment 3: Spatial separation or disparity-gradient limit?

In Experiment 1, we showed that next-to-nearest-neighbor matches occurred when the relative disparities that would have been produced by nearest-neighbor matches were greater than  $180^\circ$ . The aim of the present experiment was to determine the spatial extent of separations between center and flanking Gabors over which this interaction occurs and whether the interaction was limited by either a constant vertical separation or a constant disparity gradient (Tyler, 1973, 1974; Burt & Julesz, 1980).

#### 4.1. Method

##### 4.1.1. Stimuli and procedure

The configuration of stimuli in this experiment was the same as in Experiment 1. Minimum absolute disparities of  $90^\circ$ ,  $112.5^\circ$ ,  $135^\circ$  and  $157.5^\circ$  of 1 cpd were used. The relative disparity from nearest-neighbor matches corresponded to  $180^\circ$ ,  $225^\circ$ ,  $270^\circ$ , and  $315^\circ$ , respectively. The minimum-relative-disparity from next-to-nearest-neighbor matches for the same targets corresponded to  $180^\circ$ ,  $135^\circ$ ,  $90^\circ$  and  $45^\circ$ , respectively. Vertical separation between the edges of the flank and center Gabors was varied from  $0.125^\circ$  to  $4^\circ$ . In light of the results of Experiment 1 (Fig. 3), we chose a stimulus width that resulted in both next-to-nearest-neighbor matches for the combined center-flank condition and nearest-neighbor matches for the center-only condition at or above 75%. A sigma of  $2.5^\circ$  achieved these aims. The test stimuli and a fixation point were presented for one of two durations: 140 ms in a temporal rectangular window (transient stimulus) or 7 s in a temporal raised cosine window (sustained stimulus).

### Effect of vertical separation on transient stereo performance at 1cpd carrier spatial frequency

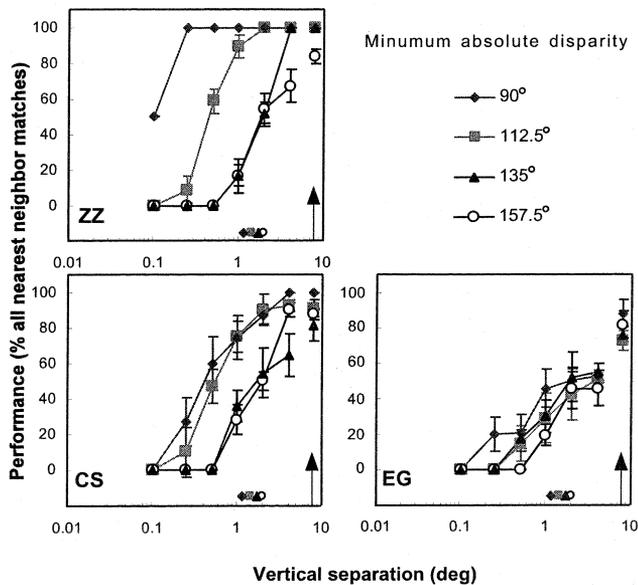


Fig. 4. Results for Experiment 3 for stimuli presented at a 140 ms duration (Transient stimulus). The carrier spatial frequency was 1 cpd; the Gaussian width was  $2.5^\circ$ . Transient stereo-performance (% all nearest-neighbor matches) is plotted against vertical separation for four different absolute disparities. Points plotted above the arrows represent the results with the center-only stimulus. The four symbols at the bottom of the figure represent the horizontal spread of the data of the four absolute disparities as predicted for a single disparity gradient limit. The results demonstrate that neither vertical separation nor a disparity gradient limit accounts for differences in performance with different absolute disparities.

#### 4.2. Results and discussion

The results for the three observers with the 1 cpd transient and sustained stimuli are shown in Figs. 4 and 5, respectively. Performance (percentage of responses corresponding to nearest-neighbor matches for all Gabors) is plotted against the vertical separation between the center and flanking Gabors. The isolated data points, plotted above the vertical arrow, represent performance for the center-only condition. Inspection of Fig. 4 (transient condition) reveals that for all observers, the percentage of next-to-nearest-neighbor matches increased as vertical separation was decreased.

If a critical vertical separation determined the absolute disparity of the binocular matches, all data would lie on one curve. Alternatively, if a constant disparity gradient determined matches (Tyler, 1973, 1974; Burt & Julesz, 1980), the curves would be separated over a small range, as noted by the four symbols at the bottom of each graph. Clearly, the responses to small and large disparity stimuli have a greater spread than predicted by the disparity gradient limit, but their sequence is in the same order. Inspection of Fig. 4 reveals that perfor-

mance as a function of Gabor separation was nearly identical for the two largest absolute disparities ( $135^\circ$  and  $157.5^\circ$ ) and was consistent across subjects. The predicted differences for these two stimuli, based upon the disparity gradient limit and the vertical separation limit, were too small to categorize the observers' responses. The percentage of nearest-neighbor matches was much higher for the two smallest absolute disparities ( $90^\circ$  and  $112.5^\circ$ ) than for the larger disparities. Nearest-neighbor matches could be made at vertical separations as small as  $0.25^\circ$  for ZZ with the  $90^\circ$  disparity stimulus, whereas he was only able to achieve a nearest-neighbor match with a  $4^\circ$  vertical separation with the two largest absolute disparity stimuli.

Relative disparity magnitudes resulting from the two matching alternatives could have influenced the results. As the minimum absolute disparity of the stimuli increased, the magnitude of relative disparities resulting from nearest-neighbor matches increased from  $180^\circ$  to  $315^\circ$  whereas the minimum relative disparities resulting from the next-to-nearest-neighbor matches decreased from  $180^\circ$  to  $45^\circ$ . Consequently, the difference between these two relative disparities increased with the magnitude of minimum absolute disparity. Given a minimum-

### Effect of vertical separation on sustained stereo performance at 1cpd carrier spatial frequency

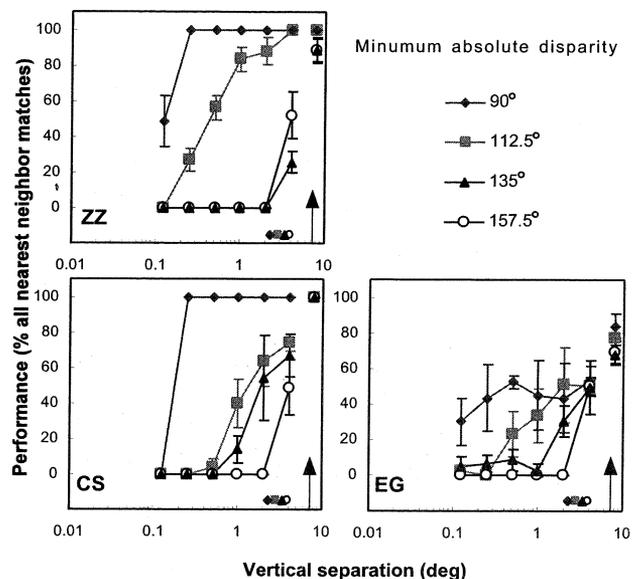


Fig. 5. Results for Experiment 3 for stimuli presented at a 7 s duration (sustained stimulus). The carrier spatial frequency was 1 cpd; the Gaussian width was  $2.5^\circ$ . Sustained stereo-performance is plotted against vertical separation for four different absolute disparities. Points plotted above the arrows represent the results with the center-only stimulus. The four symbols at the bottom of the figure represent the horizontal spread of the data of the four absolute disparities as predicted for a single disparity gradient limit. The results demonstrate that neither vertical separation nor a disparity gradient limit accounts for differences in performance with different absolute disparities.

absolute disparity of  $d$ , that is larger than  $90^\circ$ , the relative disparity resulting from a nearest-neighbor match is  $2d$ , and the minimum relative disparity resulting from a next-to-nearest-neighbor match is  $360^\circ - 2d$ . The difference in these two relative disparities is  $4d - 360^\circ$ . For our largest absolute disparity of  $157.5^\circ$ , this difference is  $270^\circ$ , while for the  $90^\circ$  stimulus, this difference is  $0^\circ$ . Thus, the benefit of using the next-to-nearest-neighbor matching solution increased as the absolute disparity of our stimulus increased. This difference between amplitudes of alternative relative disparity solutions may have influenced the higher percentage of next-to-nearest neighbor matches found with the larger than small absolute disparity stimuli.

It is interesting to note that the relative disparity for the  $90^\circ$  target is  $180^\circ$  for both the nearest-neighbor and next-to-nearest neighbor match, yet the probability of a next-to-nearest neighbor match increased with target proximity. Clearly, additional factors contributed to the differences between the matches with wide and narrow target separations. One possibility is that matches of disparities in the same direction (both crossed or uncrossed) had a higher priority at narrow than wide separations. The following experiment examined depth-direction matching biases for individual subjects.

The results for the sustained condition shown in Fig. 5 follow the same general trends as discussed for the transient system shown in Fig. 4. The most noticeable differences in responses to transient and sustained stimuli occurred with the two largest absolute disparities ( $135^\circ$  and  $157.5^\circ$ ). Next-to-nearest-neighbor matches occurred at much larger vertical separations for the sustained stimuli. For the  $135^\circ$  and  $157.5^\circ$  stimuli, performance with the transient stimuli reached chance levels for the  $2^\circ$  separation, whereas the performance with the sustained stimuli reached chance levels at  $4^\circ$  separation. This finding indicates that matches that minimize relative disparity are more likely with sustained than transient stimuli.

## 5. Experiment 4: Which Gabor undergoes the next-to-nearest-neighbor match?

The illustration in Fig. 2 of how the minimum relative disparity rule is implemented assumes arbitrarily that a next-to-nearest-neighbor match of the center stimulus (dashed bar) minimizes the relative disparity between the center and flanking Gabors. That is, the flanks underwent a nearest-neighbor match, and the center underwent a next-to-nearest-neighbor match in order to minimize the relative disparity between them. There are, however, a number of alternative stereo-matching combinations that would minimize relative disparity and lead to a perception of reversed depth ordering of center and flanking Gabors compared to

the depth ordering with all nearest-neighbor matches. These possibilities are shown in Fig. 6. They fall under three categories: subjective bias, stimulus bias, and complete reversal.

It has been noted that some individuals exhibit asymmetric stereo-depth sensitivity in response to large ( $> 1^\circ$ ) transient disparities. Sensitivity can be greater to either crossed or uncrossed disparities (Richards, 1971; Schor et al., 1998). If either the center or flank could undergo a next-to-nearest-neighbor match and the other Gabor a nearest-neighbor match, so that both matches were in the same disparity direction, the observer's depth biased could determine which Gabor underwent the next-to-nearest neighbor match. In Fig. 6, B and C represent subjective biases for matches to be made in the uncrossed and crossed directions, respectively. For example, if the observer had an uncrossed-disparity bias (e.g. ZZ and EG in the current study), then surface depth ordering would be perceived as in Fig. 6B, whereas if there were a crossed-disparity bias (e.g. CS in the current study), then surface depth ordering would be perceived as shown in Fig. 6C.

Another possibility is that the stimulus determines the depth order. If the nearest-neighbor match were always made with the center stimulus, then the pattern shown in D would be observed, while if it were made with the flank stimuli, the pattern in E would be observed. It is also possible that no nearest-neighbor matches are made, and that both depth planes undergo next-to-nearest-neighbor matching, as illustrated in F. Note that this possibility differs from the other four in that it does not minimize the relative disparity between the two depth planes, and it is inconsistent with our explanation of reverse-depth ordering.

In this experiment, we sought to identify which of the alternative depth ordering patterns our subjects observed. Because it was too difficult to estimate the perceived depth ordering with transient stimuli, only sustained stimuli were used.

### 5.1. Method

#### 5.1.1. Stimuli and procedure

The basic configuration of stimuli in this experiment was the same as in Experiment 3. A fixed stimulus width ( $2.5^\circ$  sigma) was used, and we varied the vertical separation between the center and flanking stimuli. The Gabors had a fixed spatial frequency of 1 cpd and a minimum absolute disparity of  $135^\circ$  in the uncrossed and crossed direction (relative disparity of  $270^\circ$ ). Test stimuli were presented within a 7 s temporal raised-cosine window. With reference to Fig. 6, it can be seen that F is the only condition in which the Gabors would appear to straddle the fixation point. Pilot testing indicated that this pattern was not perceived, and so it is not represented in the data analysis. We differentiated

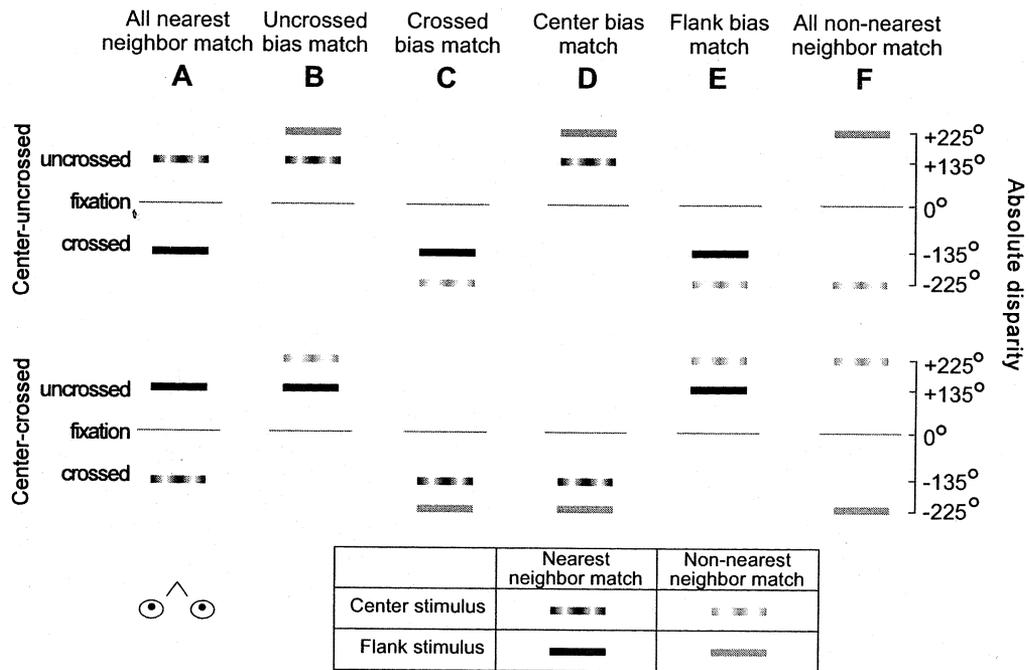


Fig. 6. Schematic illustration of six possible combinations of disparity matching solutions. The nearest-neighbor match of the center stimulus is uncrossed (far) in the top panel and crossed (near) in the bottom panel. Column A illustrates all nearest-neighbor matches. Columns B–E represent cases where either the center or flanks have a next-to-nearest-neighbor match. Columns B and C represent subject-dependent matching biases. Matches can be biased in either the uncrossed (B) or crossed (C) direction. Columns D and E represent stimulus-dependent matching biases. Matches can be biased either in the minimum absolute disparity direction of the center (D) or the flanks (E). Column F is an illustration of next-to-nearest-neighbor matches of both the center and flanks. B–F all have reversed depth ordering percepts of center relative to flank compared with the physical disparity pattern in A. Percept F was never reported.

between the five remaining conditions (A–E) by requiring observers to perform two tasks in response to each stimulus presentation. The first was to judge the depth of the center Gabor relative to the flanking Gabors. The second task was to judge the depth of the center Gabor relative to the fixation point. The results were analyzed separately for the center-crossed and center-uncrossed stimuli.

## 5.2. Results and discussion

The pattern of reverse-depth order perceived by the observers always fell into categories B or C. Each observer had a matching bias in either the crossed or uncrossed direction. Fig. 7 plots the percentage of responses made that were nearest-neighbor based, both uncrossed, or both crossed. Results of responses to the center-uncrossed condition (left-hand graphs) and center-crossed condition (right-hand graphs) are shown separately. Performance with the center Gabor in isolation (no flanking stimuli) is indicated above the vertical arrow. EG was tested at a single vertical-separation value ( $1^\circ$ ).

As found in the previous experiments, next-to-nearest-neighbor matches were most prevalent at small

stimulus separations. Each observer had the same response bias direction (crossed or uncrossed) for both the center-uncrossed and center-crossed stimuli. These biases corresponded to a single response category, either category C (all crossed matches) (CS) or B (all uncrossed matches) (ZZ & EG). Thus, the nearest-neighbor match was always made with the Gabor patch whose disparity was in the subject's bias direction.

Both sensory and motor factors can contribute to this bias. Sensory stereo-depth biases have been demonstrated in which sensitivity can be greater to either crossed or uncrossed disparities (Richards, 1971; Schor et al., 1998). There is also ample time during the 7 s of the sustained stimulus for horizontal vergence fluctuations and fixation disparity to reduce the absolute disparity of either the center or flanking stimulus and bias the response toward the minimum relative disparity match. The depth direction of the matches would be biased in the same direction as the vergence error. With transient stimuli used in the first three experiments, the stimulus duration was too short (140 ms) to allow vergence responses to change absolute disparity during the stimulus presentation, and the sensory stereo-depth asymmetry would be the main source of bias.

## 6. General discussion

The repetitive stimuli used in the current experiments allowed for a number of potential stereo matches for each Gabor patch. The minimum-absolute-disparity rule restricts matches to disparities that are less than one-half of the spatial period of the carrier (i.e. a half-cycle-limit Marr & Poggio, 1979). We found that matches beyond this half-cycle limit could be made and that they resulted in reversed perceived-depth ordering of the center and flanking Gabors. These next-to-nearest-neighbor matches reduced the relative disparity between the center and flanking Gabors, and they are consistent with a minimum-relative-disparity rule

(Mitchison & McKee, 1987a). Binocular matches increased the absolute disparity of one of the Gabors in order to minimize the relative disparity between the Gabors. The Gabor that undergoes the next-to-nearest-neighbor matching is determined by the observer's stereo-depth bias.

The present results show that next-to-nearest-neighbor matches only occur when multiple surface disparities are present. Next-to-nearest-neighbor matches do not depend upon the absolute disparity magnitude of each surface in isolation, but rather depend upon the relative disparity between the surfaces and their proximity. Specifically, next-to-nearest-neighbor matches are made when they produce a smaller relative disparity than would be obtained with all surfaces undergoing nearest-neighbor matches. The likelihood of making next-to-nearest-neighbor matches increases as either the width or spatial frequency of the stimuli is increased and also as the spatial separation between stimuli decreases. A number of our observations support these conclusions.

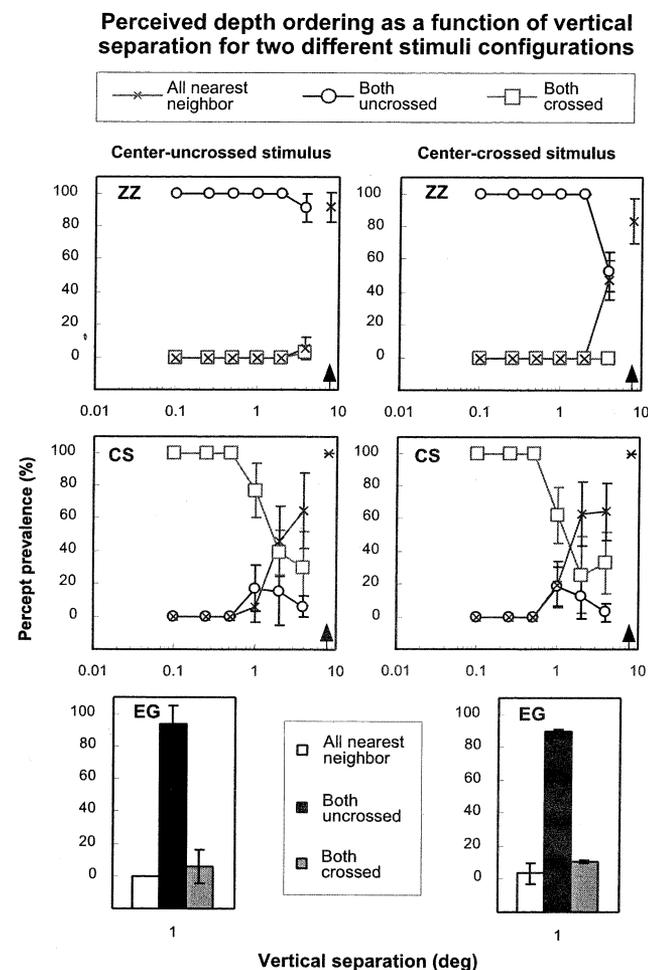


Fig. 7. Results for experiment 4 with a 7 s stimulus duration (sustained stimulus) and  $135^\circ$  minimum absolute disparity at 1 cpd. Stereo-performance is plotted as the probability of three depth percepts (all nearest neighbor, both uncrossed or both crossed) against vertical separation for center-uncrossed stimulus (left panel) and center-crossed stimulus (right panel). Points plotted above the arrows represent responses to the center-only stimulus. Subject EG was only tested with a vertical separation of 1 degree. For individual subjects, the patterns of depth matches are biased for both center-near and center-far configurations in either the crossed (CS) or uncrossed direction (ZZ & EG).

### 6.1. Relative disparity amplitude

In Experiment 1, next-to-nearest-neighbor matches were observed when the relative disparity between nearest-neighbor matches for the center and surround Gabors was greater than  $180^\circ$ . For example, next-to-nearest neighbor matches occurred when the absolute disparity of the Gabors was  $135^\circ$  (i.e. a relative disparity between nearest-neighbor matches of  $270^\circ$ ) but not with the absolute disparities of less than  $90^\circ$  (i.e. a relative disparity between nearest-neighbor matches less than  $180^\circ$ ). As can be seen from Fig. 2, this pattern of results is consistent with minimizing the relative disparity between the center and flanking stimuli. Note, however, that when we varied the vertical separation between the stimuli (Experiment 3), we observed that at very small separations ( $0.125^\circ$ ), absolute disparities of less than  $90^\circ$  could also result in consistent next-to-nearest-neighbor matches (these data are not plotted). This resulted in the center and flanks to appear in the same depth direction. This effect could be due to a form of spatial crowding that elevates stereo threshold and reduces sensitivity to small disparities (Westheimer & McKee, 1980). Crowding would bias stereo matches to larger disparities and result in next-to-nearest-neighbor matches. It is also possible that the stereo-depth direction bias described in Experiment 4 could have influenced adjacent depth matches to all appear in the same direction.

### 6.2. Absolute disparity magnitude

Experiment 2 showed that a large absolute disparity does not always result in next-to-nearest-neighbor

matches. The flanking Gabors had an absolute disparity of  $45^\circ$ , and the center Gabor had an absolute disparity of  $135^\circ$  in the same disparity direction. The finding of nearest-neighbor matches for this condition is consistent with the notion that next-to-nearest-neighbor matches are made only when they minimize relative disparity. Next-to-nearest-neighbor matches are not a general response to large absolute disparities. For this stimulus, a next-to-nearest-neighbor match for the center Gabor would have increased the relative disparity between the flank and center Gabors from  $90^\circ$  ( $135^\circ - 45^\circ$ ) to  $270^\circ$  ( $225^\circ + 45^\circ$ ).

In Experiment 2, we also showed that next-to-nearest-neighbor matches could occur with small absolute disparities. This was demonstrated by varying the relative spatial frequencies of the center and flanking Gabors. A 1 cpd center Gabor was flanked by two 0.25 cpd Gabors, and all Gabors had an absolute disparity of  $45^\circ$  (center and flanking disparities were of opposite sign). Nearest-neighbor matches for all Gabors would have resulted in a relative disparity that was greater than that obtained by making a next-to-nearest-neighbor match with the center Gabor (37.5 arc min versus 22.5 arc min).

### 6.3. Stimulus width

Experiment 1 illustrated that in the center-only condition, i.e. when there was no relative disparity to minimize, consistent next-to-nearest-neighbor matches were not made. Performance decreased as the stimulus width ( $\sigma$ ) was increased, but it stabilized at chance level (Fig. 3). We also observed chance-level performance when the wide ( $6^\circ$   $\sigma$ ) flank stimuli were presented in isolation. However, when the center and flanking stimuli of sufficient width were presented together, consistent next-to-nearest-neighbor matches were made when the relative disparity between nearest-neighbor matches for the center and surround Gabors was greater than  $180^\circ$ . Thus, wide stimuli that produced chance level performance in isolation produced consistent next-to-nearest-neighbor matches when presented together. These observations demonstrate that minimizing the relative disparity between surfaces is a powerful constraint for the stereo system that is stronger than minimizing the absolute disparity of individual features.

### 6.4. Envelope size and edges

Second-order information, provided by the envelope disparity of the Gabor patch, provides a strong cue to stereo matching of the carrier (Wilcox & Hess, 1995). Additionally, Mitchison and McKee (1987a,b) have shown that the disparity of the edges can influence the disparity matches of the features between the edges. In the present experiments, the disparity of the envelope

was in the same direction as the nearest-neighbor carrier match. Thus, the present results of next-to-nearest-neighbor matches show that even this strong envelope information can be overcome by the minimum-relative-disparity rule.

### 6.5. Stimulus separation

Experiment 3 illustrated that the likelihood of making next-to-nearest-neighbor matches decreased as vertical separation between the center and flanking Gabors increased. However, as shown in Figs. 4 and 5, there is no fixed vertical separation or disparity gradient limit that predicts performance for either the transient or sustained conditions. For a given vertical separation, the likelihood of a minimum-relative disparity solution increased with the difference between the amplitudes of alternative relative disparity matches.

### 6.6. Sustained vs. transient

Most separations where performance with the 1 cpd carrier was at the chance level ranged between  $0.5^\circ$  and  $2^\circ$  for transient stimuli and between  $0.5^\circ$  and  $4^\circ$  for sustained stimuli. Differences between responses to transient and sustained stimuli were observed with the two largest absolute disparities ( $135^\circ$  and  $157.5^\circ$ ). Next-to-nearest-neighbor matches occurred at much larger vertical separations for the sustained stimulus. This finding indicates that the sustained system is more likely than the transient system to make matches that minimize relative disparity. In contrast, McKee and Mitchison (1988) observed that the likelihood that edge disparity would influence disparity matches of points between the edges increased, as the temporal duration of the stimulus was decreased (Mitchison & McKee, 1987a,b). The different effects of exposure duration found in the two studies may depend on differences between spatial and temporal aspects of the stimuli. The stimuli differ in terms of size of both the test and inducing patterns, the meridian of separation (vertical vs. horizontal), the magnitude of the separation and the duration of the sustained stimulus. The long 7 s duration used for the sustained stimulus in the current study would provide more time than the transient stimulus for changes of horizontal vergence and absolute disparity to bias the response toward the minimum relative disparity.

### 6.7. Direction bias

Experiment 4 demonstrated that to achieve the minimum relative disparity, only the center or flank undergoes a next-to-nearest-neighbor match. This produces a disparity in the same direction as the nearest-neighbor match of the remaining stimulus. An idiosyncratic bias

determines the disparity direction of the next-to-nearest-neighbor match. Both sensory and motor factors can contribute to this bias.

### 6.8. Implementation

Our observations demonstrate that the minimum-relative-disparity rule is a powerful constraint for stereopsis, and it supersedes the minimum-absolute-disparity rule. The minimum-relative-disparity rule could be implemented by processing absolute and relative disparities serially. All possible matches for absolute disparity are represented in V1 (Cumming & Parker, 2000a,b). A subset of these matches could be selected outside area V1 to form the minimum-relative-disparity between adjacent features. Relative disparity is represented in extrastriate regions such as V2 (Thomas, Cumming, & Parker, 1999), and disparity gradients are represented in the inferior temporal cortex in area ET (Janssen, Vogels, & Orban, 2000). These areas could elaborate on absolute disparity matches represented in V1 to compute local surface percepts. We can consider the matching problem as an identification task in extrastriate cortex that selects a combination of absolute disparities to form the smallest relative disparity from a set of several alternative absolute disparity matches presented in V1. The likelihood of a minimum-relative-disparity matching solution increases when the difference between the amplitudes of the alternative relative disparities is large. When the alternative relative disparity matching solutions have similar amplitudes but have opposite signs (crossed vs. uncrossed), then a depth direction bias serves as a tie-breaker.

## 7. Conclusions

In some natural scenes, such as large, slanted, textured surfaces, the minimum absolute disparity and minimum relative disparity require different matching solutions. The current study produced a stimulus in which there was a large discrepancy in both the direction (sign) and magnitude of matches that solved for the minimum absolute and minimum relative disparity. This discrepancy has revealed a dominance of the minimum relative disparity over the minimum absolute disparity matching solution that increases with spatial proximity. In natural scenes, such as a slanted textured plane, the minimum-absolute-disparity and minimum-relative-disparity matching solutions are in conflict when disparities in regions away from the point of fixation exceed one cycle of the texture spacing. Except for the edges, most textured areas of the plane have potential matching ambiguity, and their depth solution will depend on the area, spatial separation and disparity of adjacent textured regions. Binocular matches

minimize the relative disparity between adjacent regions of the surface so that the textured surface is perceived as planar rather than fragmented (Papathomas & Julesz, 1989). The limited range of spatial interactions promotes matches that reduce abrupt changes in disparity while they allow large gradual increases in disparity of slanted surfaces.

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